

## Extending Transducer Calibration Range by Extrapolation

by LaVar Clegg

### Introduction

Force and torque transducers must be calibrated in a laboratory in order to be useful in their intended application. Applications of transducers range from basic process measurements to critical calibration of other transducers or equipment. The laboratory calibration consists of loading the transducer with known masses and lever arms, or using a comparison method where load is generated by hydraulic or pneumatic means and the transducer under test is compared to a reference transducer. In either method, the cost of calibration equipment rises rapidly with increasing capacity.

Many calibration laboratories have the means to calibrate force up to about 10,000 lbf and torque up to about 20,000 lb-in, but capability for higher ranges is scarce. In fact, there are a very limited number of laboratories in the United States that have the capability for force over 200,000 lbf and torque over 100,000 lb-in.

There has been a practice in the past by some manufacturers of transducers to calibrate a high capacity transducer at partial capacity, leaving the owner to go on hoping and guessing for the sensitivity of the upper end of the capacity. This gives rise to the concept of extrapolating the partial capacity calibration to full capacity, thereby providing the possibility for an increase in confidence in the extended range.

### The Problem

Strain gage transducers are basically linear. That is, the output follows the input at a near constant ratio. The nonlinearity is routinely measured and typically is in range of  $\pm 0.10\%FS$  or less. This provides for the ability to interpolate values between calibration points with near zero error. However, the same is not true for extrapolation, which is really estimating values that are beyond the observable range. Conventional wisdom has it, and logically so, that extrapolation is not a valid method of calibration. Extrapolating is similar to forecasting, and that idea helps one realize the liability of it.

The various methods of extrapolation are not all equal. The purpose of this paper is to explore a method that has reasonable validity when economic considerations do not permit a full capacity calibration.

### Extrapolation Methods

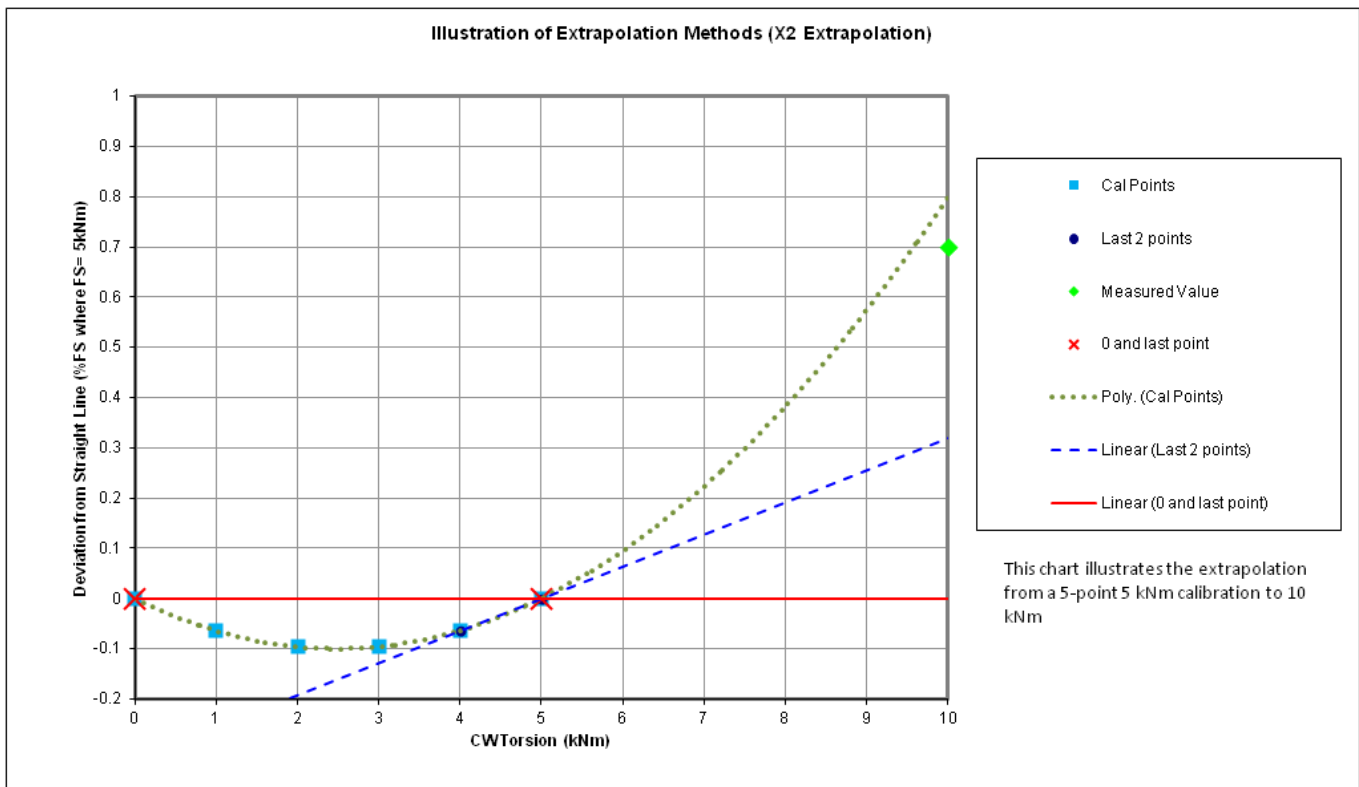


Figure 1. Extrapolation Methods

Figure 1 shows three methods of extrapolation. Given is a 5-point calibration consisting of measured values at 0%, 20%, 40%, 60%, 80%, and 100% of 5 kNm. The capacity of the transducer is 10 kNm, so it is desired to know the sensitivity between 5 and 10 kNm clockwise. The units of kNm and calibrated range of 5 are arbitrary and used only as examples. The concept applies to any range of force or torsion. Also included in the illustration is a possible measured value at capacity 10 kNm, assuming the transducer has been taken to appropriate equipment where it could be measured.

## Linear (0 and Last Point) Extrapolation

Represented by the Figure 1 solid chart line (red), this method is the most naïve and the least accurate, because it ignores the nonlinearity (there is almost always some) of the 5-point calibration. The only justification for using it at all would be if the given calibration were of only 2 points.

## Linear (Last 2 points) Extrapolation

Illustrated by the Figure 1 dashed line (blue), this is an improvement over the solid line. It does partially account for the nonlinearity of the transducer. It may be useful when no computing tools are available to make a more sophisticated estimate, as the extrapolation can be done with simple arithmetic.

## Polynomial Regression

The Figure 1 dotted curve (green) is easily the best extrapolation. It is of the form

$$Y = A_0 + A_1X + A_2X^2$$

which is a polynomial of degree 2. Higher degree polynomials were explored and almost always made poorer extrapolations than degree 2. This is somewhat intuitive, as the higher degree curves can have more radical slopes at the edge of the calibrated range. Furthermore, degree 2 behavior is expected of transducers, and higher degree fits of calibration curves are usually the result of secondary influences.

One might wonder about using a polynomial of degree 1, but that would produce a line that can never be better than the linear extrapolations explained above for a well-behaved transducer.

## Conditions for a Valid Extrapolation with Degree 2 Polynomial

Fortunately, well-designed strain gage type transducers tend to have calibration curves of a polynomial degree 2 fit. As seen in Figure 1, the extrapolation continues the smooth curve that was fitted to the known points. What is required to constitute a well-designed and well-behaved transducer? Experience with many calibrations on many varieties of transducers indicates the following guidelines for a calibration that can have a reasonably confident extrapolation (although it is not practical to quantify an uncertainty):

1. A design that avoids stress in the body anywhere significantly higher than the stress under the strain gages.
2. An integral design – joints in a transducer body make it more likely for a calibration curve fit to deviate from degree 2 and that behavior in the extrapolation region may be less well-behaved.
3. Experience with specimens of a particular model being calibrated to capacity, thus verifying the validity of extrapolation for the specimens. The more specimens that are tested the higher the confidence in extrapolations of the model.
4. Proper fixation of the transducer in the limited range calibration, including thread fits, flatness of surfaces, and torque of mounting fixture screws. Extrapolation increases the importance of the quality of the test range data.
5. Avoid the temptation to fit a polynomial of a degree higher than 2. It has been demonstrated that it is never advantageous.
6. An extrapolation multiple as low as possible – the higher the multiple, the more critical are the above guidelines. A multiple as high as 5 could be useful for some applications. A multiple of 2 is more reasonable for most.

## Uncertainty of Extrapolated Results

There is not a practical way to derive a statement of measurement uncertainty of extrapolated results. Measurement uncertainty is typically stated as an expanded uncertainty with 95% confidence. A possible method of approaching such a confidence would be to analyze a large population of a particular model or family of transducers, with calibrations over the limited range and full capacity range. This is not practical for the purposes of this paper due to cost, especially considering that specimens where extrapolation is of interest tend to be the higher capacity ones, where both the specimens and the calibration equipment are costly.

## Actual Cal Lab Results

The following force and torque transducers were calibrated in the Interface, Inc. calibration lab at both limited and full range. A degree 2 curve was then fitted to the limited range data and an extrapolated value calculated at the full range. The calculated value was then compared to the measured full capacity value.

Serial / Mode	Cal Range	Extrap Range	Extrap Ratio	Actual mV/V Extrap Range	Extrap mV/V Extrap Range	Extrap Error (%)
5634-10 CW	1200	3500	2.92	1.05036	1.05014	-0.020
5634-10 CW	1000	5000	5.00	1.50102	1.50047	-0.037
5634-10 CCW	1200	3500	2.92	-1.05036	-1.05018	-0.017
5634-10 CCW	1000	5000	5.00	-1.50102	-1.50056	-0.031
7330-13 Tension	6000	17000	2.83	1.46868	1.46887	0.013
7330-13 Tension	5000	25000	5.00	2.16010	2.16067	0.026
7330-13 Comp	6000	17000	2.83	-1.46870	-1.46888	0.012
7330-13 Comp	5000	25000	5.00	-2.16024	-2.16072	0.022
4911-12 Tension	40000	80000	2.00	1.61195	1.61155	-0.025
4911-12 Tension	40000	120000	3.00	2.42013	2.41766	-0.102
4911-12 Tension	40000	200000	5.00	4.04064	4.03057	-0.249
4911-12 Comp	40000	200000	3.00	-2.41908	-2.41858	-0.021
4911-12 Comp	40000	200000	5.00	-4.03322	-4.03222	-0.025
6439-13 CW	30000	100000	3.33	2.09807	2.09776	-0.015
6439-13 CCW	30000	100000	3.33	-2.09809	-2.09825	0.007
633667 CW	50000	100000	2.00	2.09092	2.09032	-0.029
633667 CCW	50000	100000	2.00	-2.09101	-2.09022	-0.038
633873 CW	50000	100000	2.00	2.08584	2.08651	0.032
633873 CCW	50000	100000	2.00	-2.08538	-2.08569	0.015

Figure 2 Actual Results

All of the specimens tested in this study were reported in Figure 2. The quality of the extrapolations was not known until the tests were completed, and it turned out to be reasonably pleasing. These particular transducers were not selected for their behavior but they were known to be at least typically well-behaved before conducting the tests. It cannot be concluded that extrapolations of all transducers will be this good.

## .Effect of Extrapolation Ratio

One would expect that increasing extrapolation ratio would be accompanied by increasing uncertainty of the extrapolation. We can test this argument by observing the errors reported in Figure 2. Specimen "4911-12 Tension" is particularly interesting because it was tested with a single calibration range, and then extrapolated at 3 different ratios, namely 2, 3, and 5. Figure 3 repeats the interesting tabulated data and proposes a chart with an error envelope. We take the liberty of plotting the envelope in the positive as well as the negative region, based on the observation that polarity of error over the entire population of tested specimens appears to be quite random.

Serial / Mode	Cal Range	Extrap Range	Extrap Ratio	Actual mV/V Extrap Range	Extrap mV/V Extrap Range	Error (%)
4911-12 Tension	40000	80000	2.00	1.61195	1.61155	-0.025
4911-12 Tension	40000	120000	3.00	2.42013	2.41766	-0.102
4911-12 Tension	40000	200000	5.00	4.04064	4.03057	-0.249

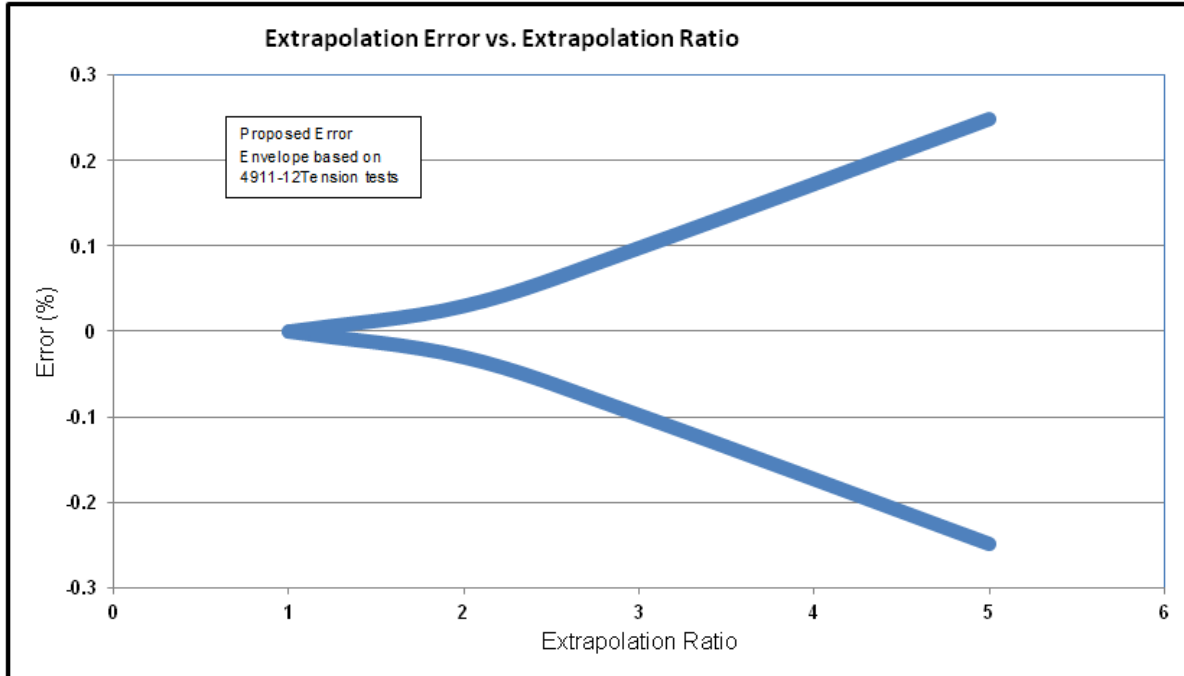


Figure 3 Extrapolation Error vs. Extrapolation Ratio

## How to Perform Polynomial Regression with Microsoft Excel

Using the regression feature of the Excel spreadsheet is a convenient way to obtain a 2nd degree polynomial with which to perform an extrapolation. An example follows.

EXAMPLE: It is desired to estimate output for input values above the calibrated range of 100 (the units are not specified because the analysis applies for any physical units as long as we are consistent). The calibration data we are given is:

Test Point	Output
0	0.00000
20	0.40399
40	0.80803
60	1.21234
80	1.61718
100	2.02250

## The Leader in Force Measurement

First, we must create a table in Excel with the given data. We must also insert a column for values of  $X^2$  for a 2nd degree solution. The table looks like this:

X	X <sup>2</sup>	Y
0	0	0.00000
20	400	0.40399
40	1600	0.80803
60	3600	1.21234
80	6400	1.61718
100	10000	2.02250

Next we engage the regression function (Note that the Analysis ToolPak add-in program must be installed in order to access regression in Excel 2007, the basis for this example).

- On the Excel header, click the Data tab.
- On the Analysis ribbon section, click "Data Analysis"
- In the pop up window select "Regression".
- A new pop up window will appear for the regression parameter input.
- Select these cells for the "Input Y Range" and these for the "Input X Range":

0.00000
0.40399
0.80803
1.21234
1.61718
2.02250

0	0
20	400
40	1600
60	3600
80	6400
100	10000

- Select any convenient cell for the output range, allowing several rows and columns after it for the SUMMARY OUTPUT table.
- The input window should now be similar to this:

The screenshot shows the 'Regression' dialog box in Excel. The 'Input' section contains 'Input Y Range' set to '\$C\$3:\$C\$8' and 'Input X Range' set to '\$A\$3:\$B\$8'. There are checkboxes for 'Labels', 'Constant is Zero', and 'Confidence Level' (set to 95%). The 'Output options' section has 'Output Range' set to '\$E\$2', with options for 'New Worksheet Ply' and 'New Workbook'. The 'Residuals' section includes checkboxes for 'Residuals', 'Standardized Residuals', 'Residual Plots', and 'Line Fit Plots'. The 'Normal Probability' section has a checkbox for 'Normal Probability Plots'. Buttons for 'OK', 'Cancel', and 'Help' are on the right.

Finally, after clicking the OK button the output will appear.

SUMMARY OUTPUT								
<i>Regression Statistics</i>								
Multiple R	0.99999999							
R Square	0.99999998							
Adjusted R Square	0.99999997							
Standard Error	0.00013011							
Observations	6							
<i>ANOVA</i>								
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>			
Regression	2	2.862902508	1.4314513	84561870	2.36252E-12			
Residual	3	5.07836E-08	1.693E-08					
Total	5	2.862902559						
	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>	<i>Lower 95.0%</i>	<i>Upper 95.0%</i>
Intercept	8.96429E-05	0.00011792	0.7602034	0.5024185	-0.00028563	0.00046492	-0.00028563	0.00046492
X Variable 1	2.01794E-02	5.54593E-06	3638.6027	4.578E-11	0.020161777	0.02019708	0.020161777	0.02019708
X Variable 2	4.39732E-07	5.32344E-08	8.2602957	0.0037156	2.70316E-07	6.0915E-07	2.70316E-07	6.0915E-07

We are interested in only the Coefficients column. From this we can write our polynomial as:

$$Y = 8.96429E-05 + 2.01794E-02 X + 4.39732E-7 X^2$$

Solving the polynomial for actual cal points and arbitrarily selected extrapolated points produces the following values. Figure 4 is a plot of the values as deviations from a straight line, and shows that the extrapolated values lie on a smooth curve along with the observed values. Figure 4 is silent regarding error in the extrapolated values because there are no observed values above 100.

Test Point	Output	Straight Line Deviation (%FS)	
0	0.00000	0	
20	0.40399	-0.025	Actual
40	0.80803	-0.048	Actual
60	1.21234	-0.057	Actual
80	1.61718	-0.041	Actual
100	2.02250	0.000	Actual
120	2.42795	0.047	Extrapolated
150	3.03690	0.156	Extrapolated
200	4.05356	0.423	Extrapolated

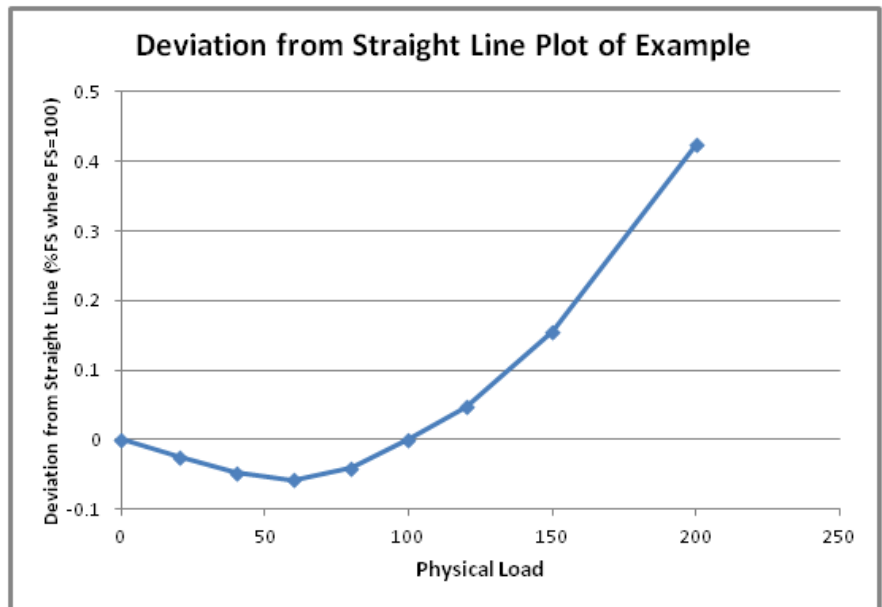


Figure 4 Plot of Extrapolation is a Smooth Curve